Prof. Frithjof Lutscher, University of Ottawa, MAT 1332, Winter 2009
Assignment 5, due March 23, 10am in class

Student Name ______________________________ Student Number ______________________________

DGD 1 (FTX 227)   DGD 2 (CBY B012)   DGD 3 (TBT 070)   DGD 4 (MCD 121)

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________________

Numbers in square brackets denote marks. No part marks will be given.

[4] 1. Calculate and put the following in standard form (i.e. in the form $z = a + ib$).

$$\text{(a) } (2+3i)(1+i) \qquad \text{(b) } |-4+2i| \qquad \text{(c) } 2\,(\overline{-4+2i}) \qquad \text{(d) } \frac{1+i}{1-i}$$

[2] 2. Put $z = 1 + 3i$ in its trigonometric form, i.e. in the form $z = |z|\,(\cos\theta + i\sin\theta)$.

[6] 3. Find the eigenvalues and the eigenvectors of $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 1 & 1 \end{bmatrix}$.

[2] 4. Find the eigenvalues of $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$.

Bonus: [2 points] Find the eigenvectors of B. [Hint: complex numbers will be involved.]

[6] 5. Let $x_{n+1} = Ax_n$ be a discrete dynamical system, with the initial state $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}$.

(a) Why is A a *Markov* matrix?

(b) Find the eigenvectors and the eigenvalues of A.

(c) Find the equilibrium point (i.e. the eigenvector associated with the eigenvalue $\lambda = 1$) by choosing the parameter such that the sum of the components of the equilibrium point is equal to 1.

(d) Write the initial state x_0 as a linear combination of the eigenvectors of A (with the equilibrium written as in c)).

(e) Write the solution x_n by calculating $x_n = A^n x_0$.